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2008 J. Phys. A: Math. Theor. 41 425401

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On scale symmetry breaking and confinement in $D = 3$ models

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Received 9 June 2008, in final form 23 July 2008

Published 22 September 2008

Online at stacks.iop.org/JPhysA/41/425401

Abstract

Within the framework of the gauge-invariant, but path-dependent, variables formalism, we study the connection between scale symmetry breaking and confinement in three-dimensional gluodynamics. We explicitly show that the static potential profile contains a linear potential, leading to the confinement of static charges. Also, we establish a new type of equivalence among different three-dimensional effective theories.

PACS numbers: 12.38.Aw, 14.80.Mz

1. Introduction

One of the long-standing issues in non-Abelian gauge theories is a quantitative description of confinement. In this context, it may be recalled that phenomenological models have been of considerable importance in our present understanding of the physics of confinement, and can be considered as effective theories of QCD. One of these, which is the dual superconductivity picture of QCD [1], has probably enjoyed the greatest popularity. In this picture, the crucial feature is the condensation of topological defects originated from quantum fluctuations (monopoles). Accordingly, the color electric flux linking quarks is squeezed into strings (flux tubes), and the non-vanishing string tension represents the proportionality constant in the linear, quark confining, potential. Lattice calculations have confirmed this picture by showing the formation of tubes of gluonic fields connecting colored charges [2]. Recently, 't Hooft [3] has suggested a new approach to the confinement problem which includes a linear term in the dielectric field that appears in the energy density. It should be highlighted at this point that QCD, at the classical level, possesses scale invariance which is broken by quantum effects. Interestingly, these effects can be described by formulating classical gluodynamics in a curved spacetime with non-vanishing cosmological constant. More precisely, an effective low-energy Lagrangian for gluodynamics which describes semi-classical vacuum fluctuations

of gluon field at large distances is obtained [4], where a dilaton coupling to gauge fields plays an essential role in this development.

On the other hand, in recent times the connection between scale symmetry breaking and confinement in terms of the gauge-invariant but path-dependent variables formalism has been developed [5, 6]. In particular, for a phenomenological model which contains both a Yang–Mills and a Born–Infeld term, we have shown the appearance of a Cornell-like potential which satisfies the 't Hooft basic criterion, after spontaneous breaking of scale invariance in both Abelian and non-Abelian cases [5]. It is worthy noting here that similar results have been obtained in the context of gluodynamics in a curved spacetime [7]. In fact, we have shown the vital role played by the massive dilaton field in triggering a linear potential, leading to the confinement of static charges. Accordingly, this picture may be considered as equivalent to that based on the condensation of topological defects. In this way, we have established a new correspondence between these two non-Abelian effective theories. The present work is aimed at studying the stability of the above scenario for the three-dimensional case. The main purpose here is to reexamine the effects of the dilaton field on a physical observable, and to check if a linearly increasing gauge potential is still present whenever we go over into three dimensions.

Before going ahead, we would like to recall a number of motivations to undertake our study $(2 + 1)$ D. In this spacetime, Yang–Mills theories are super-renormalizable and mass for the gauge fields are not in conflict with gauge symmetry [8]. Indeed, topologically massive Yang–Mills theories are a very rich field of investigation and it has been shown that Yang–Mills–Chern–Simons models are actually ultraviolet finite [9]. Yet, $(2 + 1)$ D theories may be adopted to describe the high-temperature limit of models in $(3 + 1)$ D [10]. Planar gauge theories are also of interesting to probe low-dimensional condensed matter systems, such as the description of bosonic collective excitations (like spin or pairing fluctuations) by means of effective gauge theories and high- T_C superconductivity, for which planarity is a very good approximation [11]. We should also mention that $(2 + 1)$ D theories, specially Yang–Mills theories, are very relevant for a reliable comparison between results coming from the continuum and lattice calculations, for much larger lattices can be implemented in three spacetime dimensions [12]. Most recently, 3D physics has been raising a great deal of interest in connection with branes activity; in this context, issues like self-duality [13] and new possibilities for supersymmetry breaking as induced by 3-branes [14] are of special relevance. In addition, the study of the quark–antiquark potential for some non-Abelian $(2 + 1)$ -dimensional Yang–Mills theories has been considered in [15, 16].

In order to accomplish the purpose of probing different aspects of three-dimensional field-theoretic models, we shall work out the static potential for three-dimensional gluodynamics in curved spacetime along the lines of [5, 7]. Our treatment is manifestly gauge invariant for the static potential. This analysis gives us an opportunity to compare our procedure with related three-dimensional models. As will be seen, the three-dimensional gluodynamics version is equivalent to a Lorentz- and CPT-violating Maxwell–Chern–Simons model, while a three-dimensional phenomenological model which includes a Yang–Mills and a Born–Infeld term is equivalent to the above models in the short-distance regime. At this point, we should recall that in recent times much attention has been devoted to the concept of duality by its unifying role in physics. As is well known, duality refers to an equivalence between two or more quantum field theories whose corresponding classical theories are different. Consequently, one obtains more information about a theory than is possible by considering a single description. So in this work we are primarily concerned with the physical content associated with duality. In fact, one can verify the existence of duality between two or more apparently different theories

by computing physical quantities in these theories. If the answers agree the theories should be dual.

This approach is the main focus of our paper. In this sense it should be understood our equivalence among theories. One important advantage of this approach is that it allows us to describe different models in an unified way. The point we wish to emphasize, however, is that we once again corroborate that confinement arises as an Abelian effect. In general, this picture agrees qualitatively with that of Luscher [17]. More recently, it has been related to relativistic membrane dynamics in [18], and implemented through the Abelian projection method in [19].

2. Interaction energy

We turn now to the problem of obtaining the interaction energy between static point-like sources for the three models we shall consider in this work. With this purpose, we shall compute the expectation value of the energy operator, H , in the physical state, $|\Phi\rangle$, describing the sources, which we will denote by $\langle H \rangle_\Phi$. We begin by summarizing very quickly the dilaton effective Lagrangian coupled to gluodynamics, and introduce some notation that is needed for our subsequent work. We start from the four-dimensional spacetime Lagrangian density [4],

$$\mathcal{L}^{(3+1)} = \frac{|\varepsilon_V|}{m^2} \frac{1}{2} e^{\chi/2} (\partial_\mu \chi)^2 + |\varepsilon_V| e^\chi (1 - \chi) - e^\chi (1 - \chi) \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}, \quad (1)$$

where the real scalar field (dilaton) χ , of mass m , describes quantum fluctuations, and $-|\varepsilon_V|$ is the vacuum energy density. Let us also mention here that the stable minimum is in $\chi = 0$, according to the work of [20]. Following our earlier procedure [7], we shall now consider the expansion near $\chi = 0$. In such a case, expression (1) becomes

$$\mathcal{L}_{\text{eff}}^{(3+1)} = -\frac{1}{4} F_{\mu\nu}^a \left(1 + \frac{m^2}{\Delta_{(3+1)}} \right) F^{a\mu\nu} + \frac{m^2}{32|\varepsilon_V|} (F_{\mu\nu}^a)^2 \frac{1}{\Delta_{(3+1)}} (F_{\mu\nu}^a)^2 + |\varepsilon_V|, \quad (2)$$

where $\Delta_{(3+1)}$ stands for the usual D'Alembertian in four dimensions. It is advisable to comment on the gauge invariance of the action above, after the χ -field has been integrated out. By expanding χ around $\chi = 0$, and keeping only the quadratic and linear terms in the χ -fluctuation, the standard functional integration procedure yields expression (2). Note that the χ -field is a gauge singlet, upon integration over it one should not break gauge invariance. Indeed, the combination $-\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$ appears taken at the same spacetime point and the operator $\Delta_{(3+1)}^{-1}$ acts entirely on the product $F_{\mu\nu}^a F^{a\mu\nu}$. Therefore, the gauge transformations of the field strengths are taken at the very same spacetime point, ensuring then the gauge invariance of the action terms. Here we would mention that, in view of the term

$$\int d^4x d^4y \frac{\delta^{(4)}(x-y)}{\Delta_{(3+1)}^{(x)}} F_{\mu\nu}^a(x) F^{a\mu\nu}(x), \quad (3)$$

(as before, $\Delta_{(3+1)}^{(x)}$ indicates a D'Alembertian on the x -coordinates) the integral over y yields an infrared singularity, which may readily be seen if we go over into momentum space. However, at this point, we anticipate (and it shall soon become clearer in this Section) that what we have in mind is to write an effective $(2+1)$ -dimensional non-Abelian model where one spacelike dimension is to be compactified. As it is going to become explicit same steps below, the compactification radius introduces a non-trivial pole in momentum space and removes the undesirable infrared divergence we are talking about. So, we can finally state that gauge symmetry is not lost, for $F_{\mu\nu}^a F^{a\mu\nu}$ is all taken at the same spacetime point, and the potential infrared singularity it displays is actually not present in the effective model we shall be dealing with to discuss confinement in $(2+1)$ dimensions.

Next, in order to linearize this theory, we introduce the auxiliary field, ϕ . Then, we write

$$\mathcal{L}_{\text{eff}}^{(3+1)} = -\frac{1}{4}F_{\mu\nu}^a \left(1 + \frac{m^2}{\Delta_{(3+1)}}\right) F^{a\mu\nu} + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{4}\frac{m}{\sqrt{|\varepsilon_V|}}\phi(F_{\mu\nu}^a)^2 + |\varepsilon_V|. \quad (4)$$

Once again, by expanding about $\phi = \phi_0$, we then get

$$\mathcal{L}_{\text{eff}}^{(3+1)} = -\frac{1}{4}F_{\mu\nu}^a \frac{1}{\varepsilon} \left(1 + \frac{\varepsilon m^2}{\Delta_{(3+1)}}\right) F^{a\mu\nu} + |\varepsilon_V|, \quad (5)$$

where $\frac{1}{\varepsilon} \equiv 1 + \frac{m}{\sqrt{|\varepsilon_V|}}\phi_0$. Notwithstanding, in order to set-up the context for our discussion, it is useful to recall that the field configuration ϕ_0 must be constant, so that the terms in $\dot{\phi}^2$ and $(\nabla\phi)^2$ do not add positive contributions to the energy. Actually, we must have that ϕ_0 is zero. To understand why ϕ_0 must be zero, we examine its contribution to the density energy, Θ^{00} ; with spacetime independent ϕ_0 , we have

$$\Theta^{00} = \frac{1}{2}\frac{m}{|\varepsilon_V|}\phi_0(\mathbf{E}^a \cdot \mathbf{E}^a + \mathbf{B}^a \cdot \mathbf{B}^a), \quad (6)$$

where \mathbf{E}^a and \mathbf{B}^a are respectively the electric and magnetic fields. To minimize such a term, we see that ϕ_0 must be zero and the minimum of energy turns out to be $-|\varepsilon_V|$, according to what discussed in [4]. In such a case, expression (5) reduces to

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}F_{\mu\nu}^a \left(1 + \frac{m^2}{\Delta}\right) F^{a\mu\nu} + |\varepsilon_V|. \quad (7)$$

Though, for practical purposes we could have initiated our discussion from equation (7), we believe it is worthwhile to go back to the master action, the one given in equation (1), to present our chain of considerations on the approximations which allow us to integrate over the χ -fluctuations. Also, in order to better clarify the question that the gauge invariance is not broken upon the χ -integration, we believe it is suitable to show how the field strengths come out taken at the same spacetime point.

Our immediate undertaking is to obtain the corresponding effective Lagrangian density in (2 + 1) dimensions. In other terms, this means that we have to compactify one spacelike dimension. In order to do so, we employ a sort of Kaluza–Klein approach [21], where the limit of infinite compactification radius is obtained by means of a self-consistency condition, as we shall see below. According to this idea, one writes

$$\mathcal{L}_{\text{eff}}^{(2+1)} = -\frac{1}{4}F_{\mu\nu}^a \sum_n \left(1 + \frac{m^2}{\Delta_{(2+1)} + a^2}\right) F^{a\mu\nu} + |\varepsilon_V|, \quad (8)$$

with $a^2 \equiv n^2/R^2$, and R is the compactification radius. We see, therefore, that the novel feature of the present theory is the presence of the a^2 -term. Incidentally, it is of interest to note that equation (8) shows up that the infrared divergences we have mentioned previously are avoided in virtue of the a^2 -term in the denominator. Such a question motivates us to study the role of the dilaton field in the three-dimensional case. Having characterized the new effective Lagrangian, we can now compute the interaction energy for a single mode in equation (8). To this end, we shall first examine the Hamiltonian framework for this theory. The canonical momenta are $\Pi^{a\mu} = -(1 + \frac{m^2}{\Delta+a^2})F^{a0\mu}$, which results in the usual primary constraint, $\Pi^{a0} = 0$, and $\Pi^{ai} = -(1 + \frac{m^2}{\Delta+a^2})F^{a0i}$. Here, we have simplified our notation by setting $\Delta_{(2+1)} \equiv \Delta$. This allows us to write the following canonical Hamiltonian:

$$H_C = \int d^2x \left\{ \frac{1}{2}\Pi^{ai} \left(1 + \frac{m^2}{\Delta + a^2}\right)^{-1} \Pi^{ai} + \frac{1}{4}F_{ij}^a \left(1 + \frac{m^2}{\Delta + a^2}\right) F^{aij} \right\} + \int d^2x \{ \Pi^{ai} (\partial_i A_0^a + g f^{abc} A_0^c A_i^b) \}. \quad (9)$$

The secondary constraint generated by the time preservation of the primary constraint $\Pi^{a0} \approx 0$ is now $\Gamma^{a(1)}(x) \equiv \partial_i \Pi^{ai} + g f^{abc} A^{bi} \Pi_i^c \approx 0$. It is straightforward to check that there are no more constraints and that the above constraints are first class. The corresponding extended Hamiltonian (that generates translations in time) is given by $H = H_C + \int dx (c_0^a(x) \Pi_0^a(x) + c_1^a(x) \Gamma^{a(1)}(x))$, where $c_0^a(x)$ and $c_1^a(x)$ are arbitrary multipliers. Since $\Pi^{0a} = 0$ always, and $\dot{A}_0^a(x) = [A_0^a(x), H] = c_0^a(x)$, the dynamical variables A^{0a} and their conjugate Π^{0a} may now be eliminated from the theory. We therefore drop the term in Π^{0a} and define a new arbitrary coefficient $c^a(x) = c_1^a(x) - A_0^a(x)$. The Hamiltonian then reduces to

$$H = \int d^2x \left\{ \frac{1}{2} \Pi^a \left(1 + \frac{m^2}{\Delta + a^2} \right)^{-1} \Pi^a + \frac{1}{4} F_{ij}^a \left(1 + \frac{m^2}{\Delta + a^2} \right) F^{aij} \right\} + \int d^2x \{ c^a(x) (\partial_i \Pi^{ai} + g f^{abc} A^{bi} \Pi_i^c) \}. \tag{10}$$

In order to break the gauge freedom of the theory, we introduce a gauge-fixing condition such that the full set of constraints becomes second class, so we choose

$$\Gamma^{a(2)}(x) = \int_0^1 d\lambda (x - \xi)^i A_i^{(a)}(\xi + \lambda(x - \xi)) \approx 0, \tag{11}$$

where λ ($0 \leq \lambda \leq 1$) is the parameter describing the spacelike straight path $x^i = \xi^i + \lambda(x - \xi)^i$, on a fixed time slice. Here, ξ is a fixed point (reference point), and there is no essential loss of generality if we restrict our considerations to $\xi^i = 0$. It immediately follows that the only non-trivial Dirac bracket is

$$\{A_i^a(x), \Pi^{bj}(y)\}^* = \delta^{ab} \delta_i^j \delta^{(2)}(x - y) - \int_0^1 d\lambda \left(\delta^{ab} \frac{\partial}{\partial x^i} - g f^{abc} A_i^c(x) \right) x^j \delta^{(2)}(\lambda x - y). \tag{12}$$

Now, we move on to compute the interaction energy between point-like sources in the theory under consideration, where a fermion is localized at the origin $\mathbf{0}$ and an antifermion at \mathbf{y} . As already mentioned, to do this we shall calculate the expectation value of the energy operator, H , in the physical state, $|\Phi\rangle$. From our above discussion, we see that $\langle H \rangle_\Phi$ reads

$$\langle H \rangle_\Phi = \frac{1}{2} \text{tr} \langle \Phi | \int d^2x \left\{ \Pi^a \left(1 + \frac{m^2}{\Delta + a^2} \right)^{-1} \Pi^a \right\} | \Phi \rangle + \frac{1}{4} \text{tr} \langle \Phi | \int d^2x F_{ij}^a \left(1 + \frac{m^2}{\Delta + a^2} \right) F^{aij} | \Phi \rangle. \tag{13}$$

At this stage, we recall that the physical state can be written as

$$|\Phi\rangle = \bar{\psi}(\mathbf{y}) U(\mathbf{y}, \mathbf{0}) \psi(\mathbf{0}) |0\rangle, \tag{14}$$

where $U(\mathbf{y}, \mathbf{0}) \equiv P \exp \left(ig \int_0^y dz^i A_i^a(z) T^a \right)$. As before, the line integral is along a spacelike path on a fixed time slice, P is the path-ordering prescription and $|0\rangle$ is the physical vacuum state. From the foregoing Hamiltonian structure, and since the fermions are taken to be infinitely massive (static), we then get

$$\langle H \rangle_\Phi = \langle H \rangle_0 + V^{(1)} + V^{(2)}, \tag{15}$$

where $\langle H \rangle_0 = \langle 0 | H | 0 \rangle$. The $V^{(1)}$ and $V^{(2)}$ terms are given by

$$V^{(1)} = \frac{1}{2} \text{tr} \langle \Phi | \int d^2x \Pi^{ai} \frac{\nabla^2}{\nabla^2 - M^2} \Pi^{ai} | \Phi \rangle, \tag{16}$$

$$V^{(2)} = -\frac{a^2}{2} \text{tr} \langle \Phi | \int d^2x \Pi^{ai} \frac{1}{\nabla^2 - M^2} \Pi^{ai} | \Phi \rangle, \tag{17}$$

with $M^2 \equiv a^2 + m^2$. From (12), one distinguishes an Abelian part (proportional to C_F) and a non-Abelian part (proportional to the combination $C_F C_A$) for both $V^{(1)}$ and $V^{(2)}$. After some lengthy, but straightforward manipulations, we find that, unlike to the (3 + 1)-dimensional case, the non-Abelian contribution to the $V^{(2)}$ term is zero. This then implies that, at leading order in g , the $V^{(1)}$ and $V^{(2)}$ terms are essentially Abelian. As a consequence, (16) and (17) take the form

$$V^{(1)} = -\frac{g^2}{2} \frac{1}{2} \text{tr}(T^a T^a) \int d^2x \int_0^y dz'_i \delta^{(2)}(x - z') \frac{\nabla_x^2}{\nabla_x^2 - M^2} \int_0^y dz_i \delta^{(2)}(x - z), \tag{18}$$

$$V^{(2)} = \frac{g^2}{2} \frac{a^2}{2} \text{tr}(T^a T^a) \int d^2x \int_0^y dz'_i \delta^{(2)}(x - z') \frac{1}{\nabla_x^2 - M^2} \int_0^y dz^i \delta^{(2)}(x - z). \tag{19}$$

According to our earlier procedure [22], we find that the potential for two opposite charges located at $\mathbf{0}$ and \mathbf{y} becomes

$$V = -\frac{g^2}{2\pi} C_F K_0(ML) + \frac{g^2}{4} C_F \frac{a^2}{M} L, \tag{20}$$

where $|\mathbf{y}| \equiv L$, $\text{tr}(T^a T^a) = C_F$ and $K_0(ML)$ is a modified Bessel function. Expression (20) immediately shows that a linearly increasing potential is still present in the three-dimensional case, corroborating the key role played by the dilaton field. Interestingly, it is observed that this is exactly the result obtained for $D = 3$ models of antisymmetric tensor fields that results from the condensation of topological defects as a consequence of the Julia–Toulouse mechanism [22]. In this context, it may be recalled that the existence of a confining phase for a continuum three-dimensional Abelian $U(1)$ gauge theory was first found by Polyakov [23], by including the effects due to the compactness of $U(1)$ group. We further note that for the zero mode case ($a = 0$), the confining term disappears in (20). However, from (8) we must sum over all the modes in (20). Note that the expression for the coefficient of the linear potential is given by

$$\sigma = \frac{g^2}{4} C_F \frac{1}{R} \sum_n \frac{n^2}{\sqrt{n^2 + m^2 R^2}}. \tag{21}$$

In the limit $R \rightarrow \infty$, we can see that the contributions from the low n modes are automatically suppressed. Only the higher modes ($mR \sim n$) are responsible for the finite value of σ , namely $\sigma_0 = \frac{3}{8} C_F g^2 m$. Therefore, according to expression (20) the linear piece of the potential stands and its slope is given by σ_0 .

3. Related models

Now, in order to check the consistency of our procedure, it is instructive to compare our result (20) with related three-dimensional models. To do this, we shall begin by recalling the phenomenological model studied in [5, 6, 24] which contains both a Yang–Mills and a Born–Infeld term,

$$\mathcal{L}_{\text{eff}}^{(2+1)} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{M}{2} \sqrt{-F_{\mu\nu}^a F^{a\mu\nu}}. \tag{22}$$

As was explained in [5, 6], the constant M spontaneously breaks the scale invariance. Recalling again that, by imposing spherical symmetry, the interaction energy can be exactly determined. Then, the Lagrangian density (22) becomes

$$\mathcal{L}_{\text{eff}}^{(2+1)} = -2\pi r \left\{ \frac{1}{4V} F_{\mu\nu}^a F^{a\mu\nu} + \frac{M^2}{4} \frac{V}{V-1} \right\}, \tag{23}$$

where V is an auxiliary field. Here $\mu, \nu = 0, 1$, where $x^1 \equiv r = |\mathbf{x}|$. We further observe that the quantization of this theory can be done in a similar manner to that in [5, 6]. This leads to the expectation value

$$\langle H \rangle_\Phi = \text{tr} \langle \Phi | \int d^2x \left(\frac{\Pi^{ai} \Pi^{ai}}{4\pi x} + \frac{|M|}{\sqrt{2}} \sqrt{\Pi^{ai} \Pi^{ai}} \right) | \Phi \rangle + \text{tr} \langle \Phi | \int d^2x \frac{1}{4} F_{ij}^a F^{aij} | \Phi \rangle. \quad (24)$$

Once again exploiting the previous procedure leading to (20), we find that

$$V = \frac{g^2}{4\pi} C_F \ln(\eta L) + \frac{|M|g}{\sqrt{2}} \text{tr}(v^{1a} e^1 T^a) L, \quad (25)$$

where η is a massive cutoff and, e^1 is a unit vector starting at $\mathbf{0}$ and ending at \mathbf{y} . We further note that, similarly to the previous case, confinement arose as an Abelian effect. Mention should be made, at this point, to the results obtained in [25, 26] for a $(2 + 1)$ -dimensional $SU(N)$ Yang–Mills theory

$$V(L) = \frac{g^2 C_F}{2\pi} \log(g^2 L) + \frac{7}{64\pi} g^4 C_F C_A L, \quad (26)$$

where C_F and C_A are the Casimir group factors. Hence, we see that the result (25) agrees with (26). Let us mention here that, in order to handle the square root in expression (24), we have written $\Pi^{ai} = v^a \Pi^{ai}$, where v^a is a constant vector in color space [6]. In this way, both color and Lorentz symmetries have been explicitly broken. In order to understand this connection between confinement and nonconservation of the Lorentz symmetry, we now examine a three-dimensional Lorentz and CPT violating Maxwell–Chern–Simons theory [27]. We also point out that this model was obtained after a reduction to $(2 + 1)$ dimensions of an Abelian gauge model with nonconservation of the Lorentz and CPT symmetries [28]. Thus, we have

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} M_A^2 \varphi^2 + \frac{s}{2} \varepsilon_{\mu\nu\lambda} A^\mu \partial^\nu A^\lambda - \varphi \varepsilon_{\mu\nu\lambda} v^\mu \partial^\nu A^\lambda, \quad (27)$$

where φ is a scalar field and v^μ is a constant vector. Integrating over φ , we get

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{8} \bar{v}^{\nu\lambda} F_{\nu\lambda} \frac{1}{\Delta + M_A^2} \bar{v}^{\gamma\beta} F_{\gamma\beta}, \quad (28)$$

where we have defined $\bar{v}^{\nu\lambda} \equiv \varepsilon^{\mu\nu\lambda} v_\mu$. According to our earlier procedure, the expectation value takes the form

$$\langle H \rangle_\Phi = \langle \Phi | \int d^2x \left\{ \frac{1}{2} \Pi^i \frac{\nabla^2}{\nabla^2 - \bar{M}_A^2} \Pi^i \right\} | \Phi \rangle - \frac{M_A^2}{2} \langle \Phi | \int d^2x \left\{ \Pi^i \frac{1}{\nabla^2 - \bar{M}_A^2} \Pi^i \right\} | \Phi \rangle, \quad (29)$$

where $\bar{M}_A^2 \equiv M_A^2 + \bar{v}^2$. As a consequence, the static potential is given by [22]

$$V = -\frac{g^2}{2\pi} C_F K_o(\bar{M}_A L) + \frac{g^2 M_A^2 C_F}{4\bar{M}_A} L. \quad (30)$$

The above potential profile is analogous to the one encountered in our previous analysis for gluodynamics in curved spacetime (20).

4. Final remarks

To conclude, the above connections are of interest from the point of view of providing unifications among diverse models. More interestingly, it was shown that our result (20)

agrees with that of the condensation of topological defects as a consequence of the Julia–Toulouse mechanism. However, although both approaches lead to confinement, the above analysis reveals that the mechanism of obtaining a linear potential is quite different. We stress here the role played by dilaton in yielding confinement: its mass contribute linearly to the string tension. We also draw the attention to the fact that the higher modes are the responsible for the non-trivial value of the string tension. Our explicit calculation shows that the low n modes are decoupled in the limit R going to infinity. It would be interesting to employ this analysis to fit with results coming from lattice calculations.

Acknowledgments

One of us (PG) wants to thank the Field Theory Group of the CBPF for hospitality and PCI/MCT for support. This work was supported in part by Fondecyt (Chile) grants 1080260, 1050546 and 7070105. (JH-N) expresses his gratitude to CNPq and the staff of the Department of Physics of the Universidad Tcnica Federico Santa Mara for the pleasant stay.

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